GENERAL APTITUDE

Q. No. 1 - 5 Carry One Mark Each

1. The passengers were angry _______ the airline staff about the delay.

(A) towards (B) on (C) with (D) about

Key: (C)

2. The missing number in the given sequence 343, 1331, ________, 4913 is

(A) 4096 (B) 3375 (C) 2744 (D) 2197

Key: (D)

$$\begin{array}{cccc} 343 & 1331 & 2197 & 4913 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 7^3 & 11^3 & \text{must be } 13^3 & 17^3 \end{array}$$

[Since 7, 11, 13, 17 $\rightarrow$ Prime number series].

3. Newspapers are a constant source of delight and recreation for me. The _______ trouble is that I read ______ many of them.

(A) only, too (B) only, quite (C) even, too (D) even, quite

Key: (A)

4. It takes two hours for a person X to mow the lawn. Y can mow the same lawn in four hours. How long (in minutes) will it take X and Y, if they work together to mow the lawn?

(A) 60 (B) 80 (C) 120 (D) 90

Key: (B)

X can mow the lawn $\rightarrow$ 2 hours $\Rightarrow$ X's 1 hour work $= \frac{1}{2}$

Y can mow the lawn $\rightarrow$ 4 hours $\Rightarrow$ Y's 1 hour work $= \frac{1}{4}$

$$(X+Y)'s \rightarrow 1\,\text{hour work} = \left(\frac{1}{2}+\frac{1}{4}\right) = \frac{2+1}{4} = \frac{3}{4}$$

$$\text{Total time required} = \frac{1}{\left(\frac{3}{4}\right)} = \frac{4}{3}\text{hours} = \frac{4}{3}\times 60\,\text{min} = 80\,\text{min}$$

5. I am not sure if the bus that has been booked will be able to ________ all the students.

(A) deteriorate (B) sit

(C) accommodate (D) fill

Key: (C)

Q. No. 6 - 10 Carry Two Marks Each

6. Given two sets X = {1, 2, 3} and Y = {2, 3, 4}, we construct a set Z of all possible fractions where the numerators belong to set X and the denominators belong to set Y. The product of elements having minimum and maximum values in the set Z is _________.

(A) 1/12 (B) 3/8 (C) 1/8 (D) 1/6

Key: (B)

Given two sets

$$X = \{1,2,3\} \;\&\; Y = \{2,3,4\}$$

$$\therefore Z = \left\{\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{2}{2},\frac{2}{3},\frac{2}{4},\frac{3}{2},\frac{3}{3},\frac{3}{4}\right\} \rightarrow \text{set of all possible fractions;}$$

where the numerators belong to set X and the denominators belong to set Y.

$$\Rightarrow Z = \left\{ \underset{\substack{\downarrow \\ 0.25 \\ \text{Minimum}}}{\frac{1}{4}}, \frac{1}{3},\frac{1}{2},\frac{2}{4},\frac{2}{3},\frac{3}{4},\frac{2}{2},\frac{3}{3}, \underset{\substack{\downarrow \\ 1.5 \\ \text{Maximum}}}{\frac{3}{2}} \right\}$$

$$\therefore \quad \text{The required product } = \frac{1}{4}\times\frac{3}{2} = \frac{3}{8}$$

7. Consider five people-Mita, Ganga, Rekha, Lakshmi and Sana. Ganga is taller than both Rekha and Lakshmi. Lakshmi is taller than Sana. Mita is taller than Ganga.

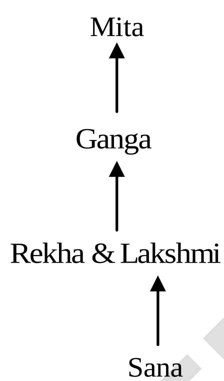
Which of the following conclusions are TRUE?

1. Lakshmi is taller than Rekha
2. Rekha is shorter than Mita
3. Rekha is taller than Sana
4. Sana is shorter than Ganga

(A) 3 only (B) 1 only (C) 2 and 4 (D) 1 and 3

Key: (C)

From the given information, we can draw as follows



Here we don't know who is taller between Rekha & Lakshmi.

So A is false:B, C & D are true.

8. How many integers are there between 100 and 1000 all of whose digits are even?

(A) 60 (B) 100 (C) 90 (D) 80

Key: (B)

Method-I:

Integers between 100 and 1000 all of the whose digits are even = 100; since

$100-199 \rightarrow$ No integers

$200-300 \rightarrow 25$ integers

[200, 202, 204, 206, 208, 220, 222, 224, 226, 228, 240, 242, 246, 248, 260, 262, 264, 266, 268, 280, 282, 284, 286, 288].

Similarly; $400-500 \rightarrow 25$ integers; $600-700 \rightarrow 25$ integers; $800-900 \rightarrow 25$ integers

$\therefore$Total number of integers$=25+25+25+25=100$

Method-II:

Required number of digits must be three.

$\therefore$ Total number of integers $= \dfrac{4}{(2,4,6,8)} \times \dfrac{5}{(0,2,4,6,8)} \times \dfrac{5}{(0,2,4,6,8)} = 100$

9. An award-winning study by a group researchers suggests that men are as prone to buying on impulse as women but women feel more guilty about shopping.

   Which one of the following statements can be inferred from the given text?

   (A) Many men and women indulge in buying on impulse

   (B) All men and women indulge in buying on impulse

   (C) Few men and women indulge in buying on impulse

   (D) Some men and women indulge in buying on impulse

Key: (D)

10. The ratio of the number of boys and girls who participated in an examination is 4:3. The total percentage of candidates who passed the examination is 80 and the percentage of girls who passed is 90. The percentage of boys who passed is _________.

    (A) 90.00 (B) 80.50 (C) 55.50 (D) 72.50

Key: (D)

Given, ratio of number of boys to girls who participated in the exam=4:3

Let, total students participated in the examination = 7x;

Given, total pass percentage = 80%

Total number of students passed in the examination $= 7x \times \dfrac{80}{100} = 5.6x$

and

The percentage of girls who passed = 90%

i.e., number of girls who passed $= 3x \times \dfrac{90}{100} = 2.7x$ [Since the number of girls participated =3x]

$\therefore$ Number of boys who passed in the exam = 5.6x – 2.7x = 2.9x

$\therefore$ Required % $= \dfrac{2.9x}{4x} \times 100 = 72.50$

ELECTRICAL ENGINEERING

Q. No. 1 to 25 Carry One Mark Each

1. Given, V_{gs} is the gate-source voltage, V_{ds} is the drain source voltage, and V_{th} is the threshold voltage of an enhancement type NMOS transistor, the conditions for transistor to be biased in saturation are

(A) $V_{gs} < V_{th}; V_{ds} \le V_{gs} - V_{th}$ (B) $V_{gs} < V_{th}; V_{ds} \ge V_{gs} - V_{th}$

(C) $V_{gs} > V_{th}; V_{ds} \le V_{gs} - V_{th}$ (D) $V_{gs} > V_{th}; V_{ds} \ge V_{gs} - V_{th}$

Key: (D)

For creating inversion layer in an n-channel MOSFET we need $V_{gs} > V_{th}$

For operating the n-channel MOSFET in the saturation region, we need $V_{dS} \ge \left(V_{gs} - V_{th}\right)$

2. The mean-square of a zero-mean random process is $\frac{kT}{C}$, where k is Boltzmann's constant, T is the absolute temperature, and C is a capacitance. The standard deviation of the random process is

(A) $\frac{\sqrt{kT}}{C}$ (B) $\frac{kT}{C}$ (C) $\frac{C}{kT}$ (D) $\sqrt{\frac{kT}{C}}$

Key: (D)

Mean square value $= \frac{kT}{C}$

Standard deviation $= \sqrt{\text{mean square value}} = \sqrt{\frac{kT}{C}}$

3. The parameter of an equivalent circuit of a three-phase induction motor affected by reducing the rms value of the supply voltage at the rate frequency is

(A) magnetizing reactance (B) rotor leakage reactance

(C) rotor resistance (D) stator resistance

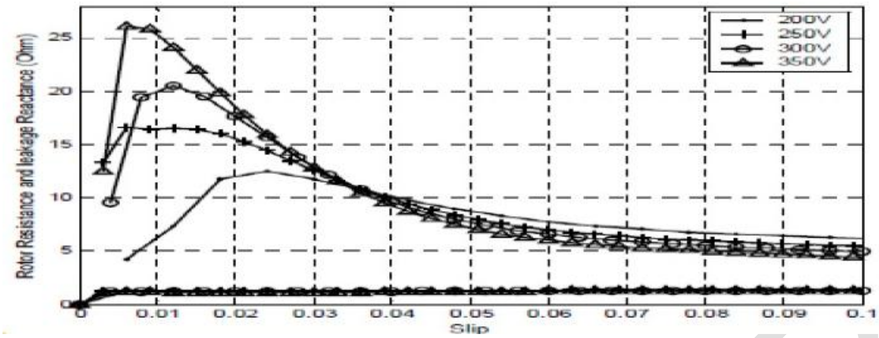
Key: (A)

NOTE:- IIT M has given answer as Magnetizing reactance (A). But above questions has multiple correct answers.

(The faculty has only thought that by changing voltage the current drawn will change and since magnetizing curve is non linear the ratio of flux linkage and current will also change and hence magnetizing reactance will changes . But if we talk about practical motor with

change in voltage the current drawn changes.this will change leakage reactance ,temperature and operating point on magnetization curve .Due to all these changes all options are correct as every parameter should change.)

Variations in XI due to supply voltage variation



Rotor leakage reactance, (top curves), and rotor resistance, (bottom curves), variations with supply voltage and low slips at a motor temperature of 40°C – single –cage machine.

$$\downarrow I = \frac{V\downarrow}{z}$$

When voltage alone reduces, the current drawn by Induction Motor reduces, Torque reduces

$$\downarrow \boxed{T_{em} \propto V^2} \downarrow$$

As the torque reduces, slip increases to get steady state operation.

$$\uparrow T_{em} \propto \frac{\uparrow sV^2}{R_2}$$

The change in slip causes change in reactance of rotor

$$\because X_{2r} = sX_2$$

4. A co-axial cylindrical capacitor show in Figure (i) has dielectric with relative permittivity $\varepsilon_{r1} = 2$. When one-fourth portion of the dielectric is replaced with another dielectric of relative permittivity ε_{r2}, as shown in Figure (ii), the capacitance is doubled. The value of ε_{r2} is __________.

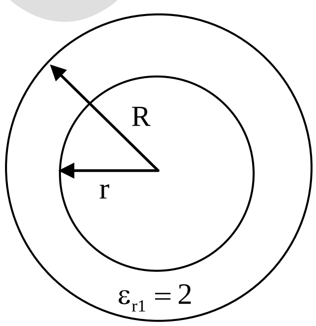


Figure (i)

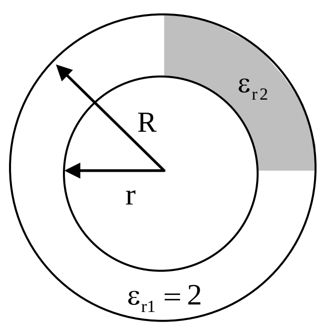
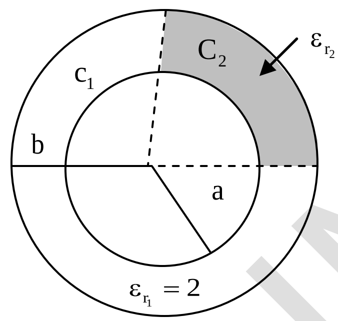


Figure (ii)

Key: (10)

The capacitance of a coaxial cable

$$C_o = \frac{2\pi\varepsilon\ell}{\ell n\left(b/a\right)} = \frac{2\pi\varepsilon_o\varepsilon_r\ell}{\ell n\left(b/a\right)} = \frac{2\pi\left(2\varepsilon_o\right)\ell}{\ell n\left(b/a\right)}$$



The two capacitors C1 & C2 are connected in parallel

$$\therefore \quad C_{eq} = C_1 + C_2$$

$$= \frac{3\left(2\pi\varepsilon_o\ell\right)}{2\ell n\left(b/a\right)} + \frac{\pi\varepsilon_o\varepsilon_{r_2}\ell}{2\ell n\left(b/a\right)}$$

Given $C_{eq} = 2C_o$

$$\frac{3\pi\varepsilon_o}{\ell n\left(b/a\right)} + \frac{\pi\varepsilon_o\varepsilon_{r_2}}{2\ell n\left(b/a\right)} = 2\left[\frac{2\pi\left(2\in_o\right)}{\ell n\left(b/a\right)}\right]$$

$$3 + \frac{\varepsilon_{r_2}}{2} = 8 \Rightarrow \varepsilon_{r_2} = 10$$

5. The output voltage of a single-phase full bridge voltage source inverter is controlled by unipolar PWM with one pulse per half cycle. For the fundamental rms component of output voltage to be 75% of DC voltage, the required pulse width in degree (round off up to one decimal place) is ___________.

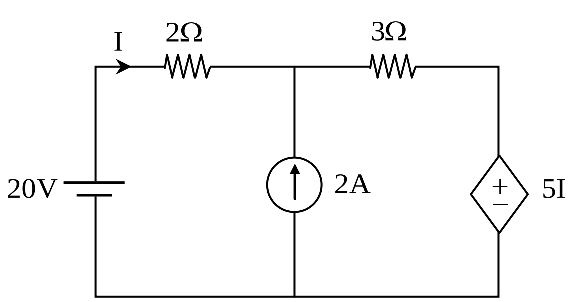
Key: (112.8)

$$0.75\ V_{dc} = \frac{4V_{dc}}{\pi\sqrt{2}}\sin\left(d\right)$$

$$\Rightarrow d = 56.41^\circ$$

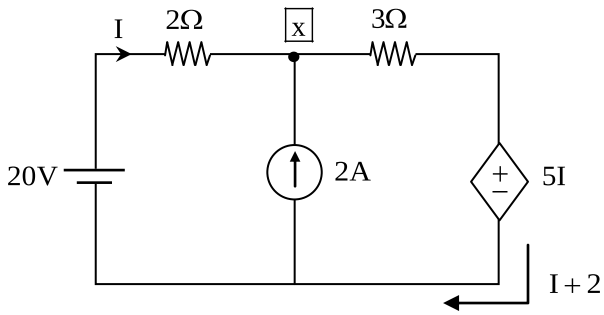
Hence the required pulse width in degrees = 2d = 2 × 56.41 = 112.82°

6. The current I flowing in the circuit shown below in amperes (round off to one decimal place) is ___________.



Key: (1.4)

The given circuit is



By KCL at node x, the current through the dependent source is I + 2.

Writing KVL at outer loop

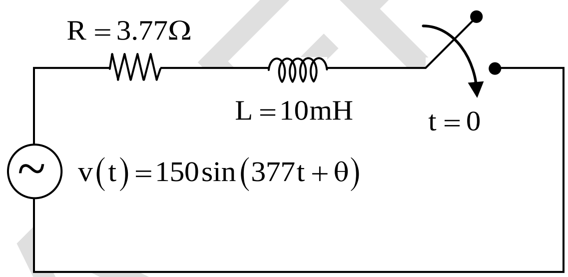
$20-2I-3(I+2)-5I=0$

$20-2I-3I-6-5I=0$

$\Rightarrow 14=10I$

$\Rightarrow I=1.4A$

7. In the circuit shown below, the switch is closed at t = 0. The value of θ in degrees which will give the maximum value of DC offset of the current at the time of switching is



(A) –45 (B) 90 (C) 60 (D) –30

Key: (A)

Sol: **Method-I:**

$i(t)=\frac{V_M}{z}\sin(\omega t+\theta-\phi)+Ae^{-t/z}$

$|Z|=\sqrt{R^2+(\omega L)^2}$

$\phi=\tan^{-1}\frac{\omega L}{R}$

At $t=0, i=0$

So, $A=\frac{-V}{|Z|}\sin(\theta-\phi)$

$$\therefore \; i(t) = \frac{V_M}{|Z|}\sin(\omega t + \theta - \phi) - \frac{V_M}{|Z|}\sin(\theta - \phi)e^{-t/Z} \Rightarrow \text{dc offset value}$$

For dc offset to be maximum

$$\sin(\theta - \phi) = \pm 1$$

$$\theta - \phi = \pm 90°$$

$$\theta = \pm 90° + \tan^{-1}\left(\frac{\omega L}{R}\right) = \pm 90° + 45°$$

$$\theta = -45° \text{ or } 135°$$

Method –II:

Electrical student can solve this question using power electronic concept.

dc offset maximum at $\theta = \pm 90° + (\text{pf angle}) = \pm 90 + 45° = -45° \text{ or } 135°$

We have option as –45° only. Hence correct answer is –45°.

8. The total impedance of the secondary winding, leads, and burden of a 5ACT is 0.01Ω. If the fault current is 20 times the rated primary current of the CT, the VA output of the CT is ___________.

Key: (100)

Sol: The VA output of the CT is $(5 \times 20)^2 \times 0.01 = 100\text{ VA}$

9. A 5kVA, 50 V/100V, single-phase transformer has a secondary terminal voltage of 95V when loaded. The regulation of the transformer is

(A) 5% (B) 9% (C) 4.5% (D) 1%

Key: (A)

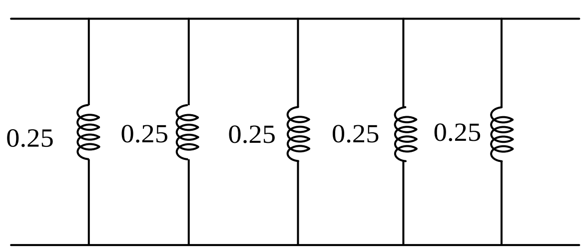
Sol: Percentage regulation of Transformer $= \frac{E_2 - V_2}{E_2} \times 100 = \frac{100 - 95}{100} \times 100 = \frac{5}{100} = 5\%$

10. Five alternators each rated 5MVA, 13.2 kV with 25% of reactance on its own base are connected in parallel to a busbar. The short-circuit level in MVA at the busbar is ___________.

Key: (100)

Sol: $$\text{S.C.MVA} = \frac{\text{Base MVA}}{X_d''}$$

$$= \frac{5}{0.25/5} = 100\text{ MVA}$$



11. A six pulse thyristor bridge rectifier is connected to a balanced three-phase, 50 Hz AC source. Assuming that the DC output current of the rectifier is constant, the lowest harmonic component in the AC input current is

(A) 100 Hz (B) 150 Hz (C) 250 Hz (D) 300 Hz

Key: (C)

Sol: The lowest Harmonic content in the AC input current is 5^{th} harmonics.

$f = 5 \times f_s = 50 \times 5 = 250$ Hz

12. The characteristic equation of a linear time-invariant (LTI) system is given by

$\Delta(s) = s^4 + 3s^3 + 3s^2 + s + k = 0$

The system BIBO stable if

(A) $k > 3$ (B) $0 < k < \frac{8}{9}$ (C) $0 < k < \frac{12}{9}$ (D) $k > 6$

Key: (B)

Sol: The given characteristic equation of LTI system is

$\Delta(s) = s^4 + 3s^3 + 3s^2 + s + k$, for

BIBO stability, we prefer R-H criterion.

$$\begin{array}{c|ccc} s^4 & 1 & 3 & k \\ s^3 & 3 & 1 & 0 \\ s^2 & 8/3 & k & \\ s^1 & \frac{8/3 - 3k}{8/3} & & \\ s^0 & k & & \end{array}$$

For stability all elements of 1^{st} column should be positive

$$\left.\begin{array}{l} \dfrac{\frac{8}{3} - 3k}{8/3} > 0 \\ \Rightarrow \frac{8}{3} > 3k \\ \Rightarrow 3k < \frac{8}{3} \\ \Rightarrow k < \frac{8}{9} \end{array}\right| \text{and } k > 0$$

$$(k > 0) \wedge \left(k < \frac{8}{9}\right) = 0 < k < \frac{8}{9}$$

13. A system transfer function is $H(s)=\frac{a_1s^2+b_1s+c_1}{a_2s^2+b_2s+c_2}$. If $a_1=b_1=0$, and all other coefficients are positive, the transfer function represents a

(A) high pass filter
(B) notch filter
(C) low pass filter
(D) band pass filter

Key: (C)

Sol: It is given that

$$H(s)=\frac{a_1s^2+b_1s+c_1}{a_2s^2+b_2s+c_2}$$

If $a_1=b_1=0$, then $H(s)$ becomes

$$H(s)=\frac{c_1}{a_2s^2+b_2s+c_2}$$

$$H(0)=\frac{c_1}{c_2}(\text{i.e., as low frequency } s\to 0\Rightarrow\omega\to 0)$$

$$H(\infty)=0(\text{i.e., as high frequency } s\to\infty\Rightarrow\omega\to\infty)$$

So the system passes low frequency and blocks high frequency. So it represents a low pass filter.

14. The symbols, a and T, represent positive quantities, and u(t) is the unit step function. Which one of the following impulse response is NOT the output of a causal linear time-invariant system?

(A) $e^{-a(t+T)}u(t)$
(B) $1+e^{-at}u(t)$
(C) $e^{-a(t-T)}u(t)$
(D) $e^{+at}u(t)$

Key: (B)

Sol: If a L.T.I system is causal, we must should have the condition.

$$h(t)=0;\text{ for } t<0$$

If we see the options a, c, d in these u(t) is multiplied that means their impulse response are zero for negative value of time, hence they are causal.

If we check option B

$$h(t)=1+e^{-at}u(t)$$

We can see h(t) = 1; t < 0, hence it represents a non causal system.

15. If $f = 2x^3 + 3y^2 + 4z,$ the value of line integral $\int_C \text{grad}\, f.dr$ evaluated over contour C formed by the segments $(-3,-3,2) \to (2,-3,2) \to (2,6,2) \to (2,6,-1)$ is __________.

Key: (139)

Sol: Given,

$f = 2x^3 + 3y^2 + 4z$

$\Rightarrow \nabla f = \hat{i}\left[6x^2\right] + \hat{j}[6y] + \hat{k}[4]$

$\therefore \quad \text{Curl } [\nabla f] = \hat{0}$

$\Rightarrow \nabla f$ is irrotational

$\Rightarrow$ The value of line integral does not dependent on the path of integration, only depends on the end points of integral.

$$\therefore \quad \int_C \text{grad} f.dr = \int 6x^2 dx + 6y dy + 4dz = \int_C d\left(2x^3 + 3y^2 + 4z\right)$$

$$= \int_{(-3,-3,2)}^{(2,-3,2)} d\left(2x^3 + 3y^2 + 4z\right) + \int_{(2,-3,2)}^{(2,6,2)} d\left(2x^3 + 3y^2 + 4z\right) + \int_{(2,6,2)}^{(2,6,-1)} d\left(2x^3 + 3y^2 + 4z\right)$$

$$\Rightarrow \int_C \text{grad}\, f.dr = \int_{(-3,-3,2)}^{(2,6,-1)} d\left(2x^3 + 3y^2 + 4z\right)$$

$$= \left[2x^3 + 3y^2 + 4z\right]_{(-3,-3,2)}^{(2,6,-1)} = 120 - (-19) = 139$$

16. A three-phase synchronous motor draws 200 A from the line at unity power factor at rated load. Considering the same line voltage and load, the line current at a power factor of 0.5 leading is

(A) 100A (B) 300A (C) 400A (D) 200A

Key: (C)

Sol: Power drawn by load $(3-\phi).P = \sqrt{3} V_L I_L \cos\phi$

$V_L = V_{rated}; \quad I_L = 200A$

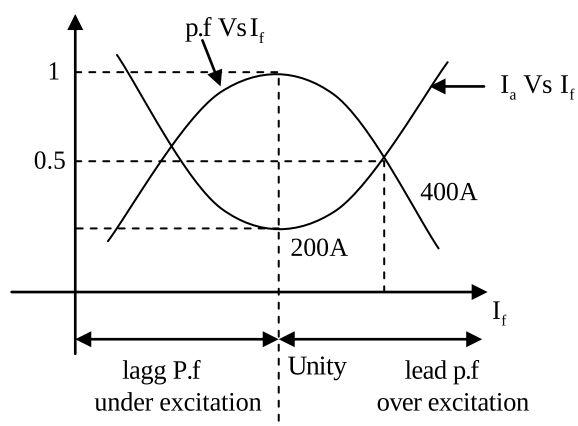
$\cos\phi = \text{unity}$

If the power factor reduces to 0.5 lead

whereas

P, V_L are same as earlier

$\uparrow I \propto \dfrac{1}{\cos\phi \downarrow} \qquad \therefore I = 400A$



17. The inverse Laplace transform of $H(s)=\dfrac{s+3}{s^2+2s+1}$ for $t\ge 0$ is

(A) $3te^{-t}+e^{-t}$ (B) $3e^{-t}$ (C) $4te^{-t}+e^{-t}$ (D) $2te^{-t}+e^{-t}$

Key: (D)

Sol: Given, $H(s)=\dfrac{s+3}{s^2+2s+1}$ for $t\ge 0$

$$\Rightarrow \quad L^{-1}\left[H(s)\right]=L^{-1}\left[\frac{s+3}{s^2+2s+1}\right]=L^{-1}\left[\frac{s+3}{(s+1)^2}\right]$$

$$=L^{-1}\left[\frac{s+1+2}{(s+1)^2}\right]=L^{-1}\left[\frac{1}{s+1}+\frac{2}{(s+1)^2}\right]$$

$$=L^{-1}\left[\frac{1}{s+1}\right]+2L^{-1}\left[\frac{1}{(s+1)^2}\right]=e^{-t}(1)+2e^{-t}t \qquad \left[\because L^{-1}\left[\frac{1}{s^2}\right]=t\right]$$

$$\Rightarrow \quad L^{-1}\left[H(s)\right]=2te^{-t}+e^{-t}$$

18. The open loop transfer function of a unity feedback system is given by

$$G(s)=\frac{\pi e^{-0.25s}}{s},$$

In G(s) plane, the Nyquist plot of G(s) passes through the negative real axis at the point.

(A) $(-1.5, j0)$ (B) $(-0.5, j0)$ (C) $(-0.75, j0)$ (D) $(-1.25, j0)$

Key: (B)

Sol: It is given that $G(s)=\dfrac{\pi e^{-0.25s}}{s}$, when the Nyquist Plot cuts the negative real axis,

its phase becomes –180°

$$\Rightarrow -180^\circ=-90^\circ-0.25\omega\frac{180^\circ}{\pi}$$

$$\Rightarrow 0.25\omega\frac{180^\circ}{\pi}=90^\circ$$

$$\Rightarrow \frac{\omega}{\pi}=\frac{0.5}{0.25}\Rightarrow \omega=2\pi$$

Magnitude at this frequency $\quad G(2\pi)=\left|\dfrac{\pi e^{-0.25j(2\pi)}}{j2\pi}\right|=\dfrac{1}{2}=0.5$

At negative real axis the co-ordinate becomes (–0.5, j0)

19. The partial differential equation $\frac{\partial^2 u}{\partial t^2} - C^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = 0$; where $C \neq 0$ is known as

(A) Wave equation (B) Poisson's equation

(C) Laplace equation (D) Heat equation

Key: (A)

Sol: Given partial D.E

$$\frac{\partial^2 u}{\partial t^2} - C^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = 0, \text{ where } C \neq 0$$

$\Rightarrow \frac{\partial^2 u}{\partial t^2} = C^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$; which is clearly two dimensional wave equation.

20. M is a 2 × 2 matrix with eigen values 4 and 9. The eigen values of M^2 are

(A) 16 and 81 (B) 2 and 3 (C) –2 and –3 (D) 4 and 9

Key: (A)

Sol: Given,

M is a 2 × 2 matrix with eigen values 4 and 9.

$\therefore$ i.e., $\lambda = 4, 9$ [Where 'λ' represents eigen values of M]

From the properties of eigen values; we have if λ is an eigen value of the matrix M then λ^2 is an eigen value of the matrix M^2.

$\therefore \lambda^2 = 4^2, 9^2 = 16, 81$ are eigen values of matrix M^2.

21. The Y_{bus} matrix of a two-bus power system having two identical parallel lines connected between them in pu is given as $Y_{bus} = \begin{bmatrix} -j8 & j20 \\ j20 & -j8 \end{bmatrix}$.

The magnitude of the series reactance of each line in pu (round off up to one decimal) place) is ____________.

Key: (0.1)

Sol: Given $Y_{bus} = \begin{bmatrix} -j8 & j20 \\ j20 & -j8 \end{bmatrix}$

$$Y(\text{each line}) = \frac{j20}{2} = j10$$

The magnitude of the series reactance of each line in $Pu = \left|\frac{1}{j10}\right| = \frac{1}{10} = 0.1pu$

22. The rank of the matrix, $M = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, is ________.

Key: (3)

Sol: Method-I

Given,

$$M\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Applying $R_3 \to R_3 - R_2$; then

$$M \sim \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

Applying

$R_3 \to R_3 - R_1$ then

$$M \sim \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & -2 \end{bmatrix}$$

Applying $R_2 \leftrightarrow R_3$;

$$M \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{bmatrix} \to \text{Echelon form of the matrix}$$

$\therefore \rho(M) = \text{Number of non-zeros in echelonform}$

$\Rightarrow \rho(M) = 3$

Method-2

$$|M| = \begin{vmatrix} + & - & + \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = -1[0-1] + 1[1-0]$$

$= 1 + 1 = 2 \neq 0.$

$\Rightarrow |M_{3\times3}| \neq 0 \Rightarrow \rho(M) = 3$

23. The output response of a system is denoted as y(t), and its Laplace transform is given by $Y(s)=\dfrac{10}{s\left(s^2+s+100\sqrt{2}\right)}$. The steady state value of y(t) is

(A) $\dfrac{1}{100\sqrt{2}}$ (B) $10\sqrt{2}$ (C) $\dfrac{1}{10\sqrt{2}}$ (D) $100\sqrt{2}$

Key: (C)

Sol: → It is given that

$$Y(s)=\frac{10}{s\left(s^2+s+100\sqrt{2}\right)}$$

We need to find steady state value of $y(t)$ i.e, $y(\infty)$

→ By final value theorem

$$y(\infty)=\lim_{s\to 0} sY(s)=\lim_{s\to 0} s\frac{10}{s\left(s^2+s+100\sqrt{2}\right)}$$

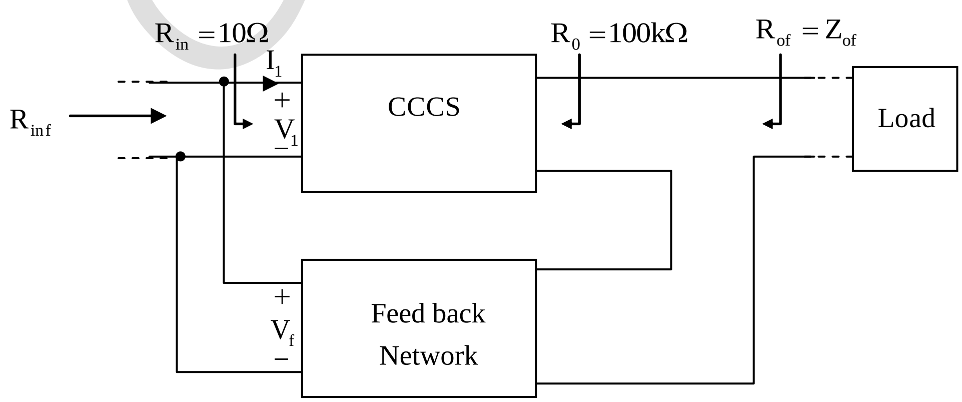
$$=\frac{10}{100\sqrt{2}}=\frac{1}{10\sqrt{2}}$$

24. A current controlled current source (CCCS) has an input impedance of 10Ω and output impedance of 100 kΩ. When this CCCS is used in a negative feedback closed loop with a loop gain of 9, the closed loop output impedance is

(A) $100\,k\Omega$ (B) $100\,\Omega$ (C) 10Ω (D) $1000\,k\Omega$

Key: (D)

Sol: For a current controlled current source, both input as well output signals are in current form. so, the correct feedback technique will be current shunt feedback (or in another word it is called as shunt series feedback)



Output impedance given loop gain $AB = 9,\ Z_0 = 100k\Omega$

$Z_{of} = Z_0(1 + AB) = 100k\Omega[1+9] = 1000k\Omega$

25. Which one of the following functions is analytic in the region $|z| \le 1$?

(A) $\frac{z^2 - 1}{z + j0.5}$ (B) $\frac{z^2 - 1}{z + 2}$ (C) $\frac{z^2 - 1}{z - 0.5}$ (D) $\frac{z^2 - 1}{z}$

Key: (B)

Sol: Given region is $|z| \le 1$; which represents the region inside and on the unit circle $|z| = 1$

The functions given in the options A, C, D are not analytic functions in the region $|z| \le 1$; Since the singular points –j(0.5), 0.5, 0 lies inside $|z| = 1$.

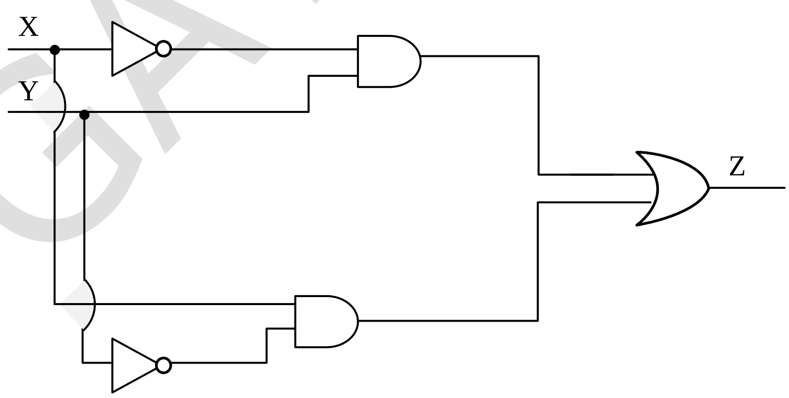
Let $f(z) = \frac{z^2 - 1}{z + 2}$

$\Rightarrow z = -2$ is the singular point but this is lies outside $|z| = 1$

$\therefore \frac{z^2 - 1}{z + 2}$ is analytic in the region $|z| \le 1$.

Q. No. 26 - 55 Carry Two Marks Each

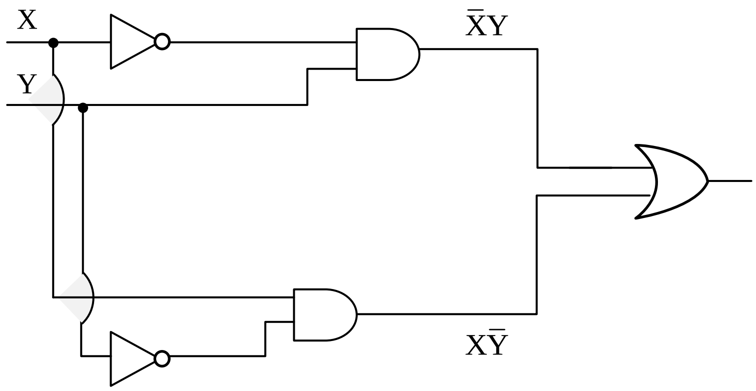
26. In the circuit shown below, X and Y are digital inputs, and Z is a digital output. The equivalent circuit is a



(A) XOR gate (B) NOR gate (C) XNOR gate (D) NAND gate

Key: (A)

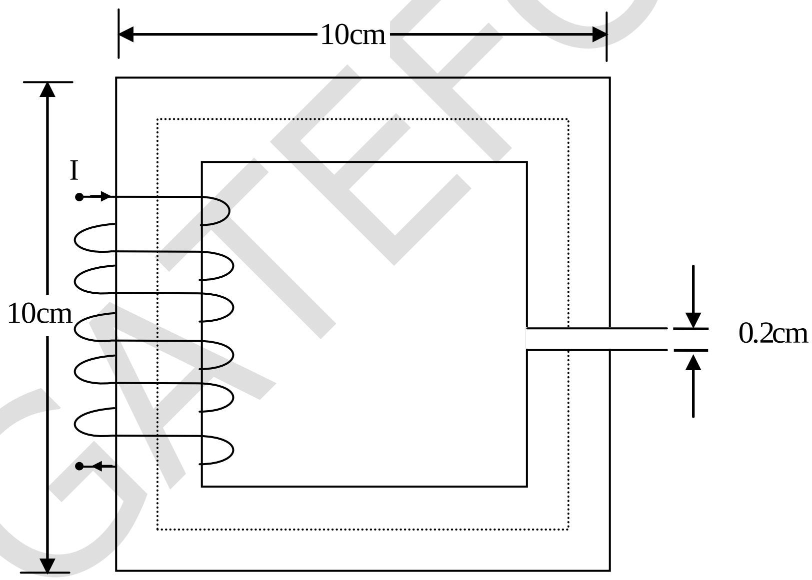
Sol:



$Z = \bar{X}Y + X\bar{Y} = X \oplus Y$

Given circuit is equivalent to XOR gate.

27. The magnetic circuit shown below has uniform cross-sectional area and air gap of 0.2 cm. The mean path length of the core is 40 cm. Assume that leakage and fringing fluxes are negligible.



When the core relative permeability is assumed to be infinite, the magnetic flux density computed in the air gap is 1tesla. With same Ampere-turns, if the core relative permeability is assumed to be 1000 (linear), the flux density in tesla (round off to three decimal places) calculated in the air gap is________.

Key: (0.834)

Sol: Given, Air gap $\left(\ell_g\right) = 0.2$ cm.

Mean length of core = 40. 0.2=39. 8 cm. (ℓ_c)

$\mu_{r_2}\left(\text{core}\right) = 1000, \mu_{r_1}\left(\text{core}\right) = \infty.$

$B_1 = 1\,T$

from $NI = \text{constant} = B.A\ \text{Reluctance}$

$$\Rightarrow B_1 A\left\{\frac{\ell_g}{\mu_0 A} + \frac{\ell_c}{\mu_0 \mu_{r_1} A}\right\} = B_2 A\left[\frac{\ell_g}{\mu_0 A} + \frac{\ell_c}{\mu_0 \mu_{r_2} A}\right]$$

$$\because \mu_{r_1} = \infty, \text{Hence } \frac{\ell_c}{\mu_0 \mu_{r_1}} \approx 0.$$

$$B_1\left\{\frac{0.2}{\mu_0}\right\} = B_2\left\{\frac{0.2}{\mu} + \frac{39.8}{100\,\mu}\right\}$$

$$B_2 = \frac{B_1(0.2)}{\left(0.2 + \frac{39.8}{1000}\right)} = \frac{1\times 0.2}{0.2398} = 0.83402\,T.$$

Hence, if the core relative permeability is assumed to be 1000, then the flux density in the air gap is 0.834 T.

28. A delta-connected, 3.7 kW, 400 V(line), three-phase, 4-pole, 50-Hz squirrel-cage induction motor has the following equivalent circuit parameter per phase referred to the stator:

$R_1 = 5.39\Omega, R_2 = 5.72\Omega, X_1 = X_2 = 8.22\Omega.$ Neglect shunt branch in the equivalent circuit. The starting line current in amperes (round off to two decimal places) when it is connected to a 100V (line), 10 Hz, three-phase AC source is__________.

Key: (14.94)

Sol: As frequency has changed to 10 Hz. Reactance will change but resistance will be same.

$$X_1 = X_2\left(\text{at } 10\text{Hz}\right) = \frac{8.22}{50}\times 10 = 1.644\Omega$$

$R_1 = 5.39\Omega\ R_2 = 5.72\Omega$

$$\text{I starting}\left(\text{line}\right)\text{at } 10\text{Hz} = \frac{100\sqrt{3}}{\sqrt{\left(5.39+5.72\right)^2 + \left(1.644\times 2\right)^2}} = 14.94\text{A}$$

29. If $A = 2xi + 3yj + 4zk$ and $u = x^2 + y^2 + z^2$, then div(uA) at (1, 1,1) is_______.

Key: (45)

Sol: Given ,

$A = 2x\,i + 3y\,j + 4z\,k$ and

$u = x^2 + y^2 + z^2$

$\text{div}(uA) = \nabla.(uA)$

$\Rightarrow \text{div}(uA) = u(\nabla.A) + \nabla u.A \,(\because \text{Using vector identities})....(1)$

$$\nabla.A = \frac{\partial}{\partial x}(2x) + \frac{\partial}{\partial y}(3y) + \frac{\partial}{\partial z}(4z)$$

$$= 2 + 3 + 4 = 9$$

$\Rightarrow \nabla.A = 9 \to (2)$

&

$$\nabla u = i\frac{\partial u}{\partial x} + j\frac{\partial u}{\partial y} + k\frac{\partial u}{\partial z};\ \text{where } u = x^2 + y^2 + z^2$$

$$= i[2x] + j[2y] + k[2z] = 2[xi + yi + zk]$$

$\therefore \nabla u.\,A = [(2x)i + (2y)j + (2z)k].[(2x)i + (3y)j + (4z)k]$

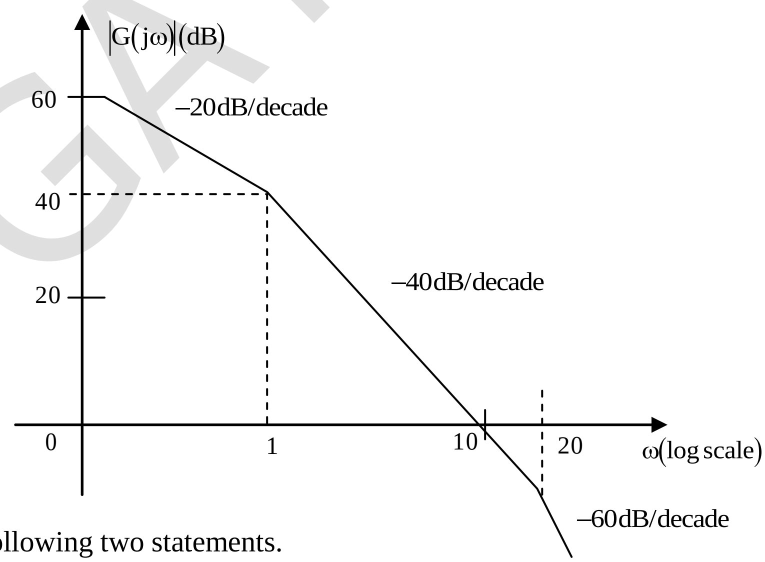
$\Rightarrow \nabla u.A = 4x^2 + 6y^2 + 8z^2 \qquad ...(3)$

From $(1),(2)\,\&\,(3)$, we have

$\text{div}(uA) = (x^2 + y^2 + z^2)9 + (4x^2 + 6y^2 + 8z^2)$

$\text{div}(uA)/_{(1,1,1)} = (1+1+1)9 + (4+6+8) = 27 + 18 = 45.$

30. The asymptotic Bode magnitude plot of a minimum phase transfer function G(s) is shown below.



Consider the following two statements.

Statement I: Transfer function G(s) has three poles and one zero.

Statement II: At very high frequency $(\omega \to \infty)$, the phase angle $\angle G(j\omega) = -\frac{3\pi}{2}$.

Which one of the following option is correct?

(A) Statement I is false and statement II is true.

(B) Both the statements are true.

(C) Both the statements are false.

(D) Statement I is true and statement II is false.

Key: (A)

Sol: → From the given bode-plot, we can say

→ At origin, there is a pole at origin, since the initial slope is -20db/dec.

→ At $\omega = 1$, the change in slope is $-40-(-20) = -20$db/sec, so it imply one pole at $\omega = 1$.

→ At $\omega = 20$, the change in slope is $-60-(-40) = -20$db/dec, so it imply one pole at $\omega = 20$.

→ So in total the transfer function has 3 poles, hence at $\omega = \infty$, the net phase contributed by 3 poles is $-270°$ or $-\frac{3x}{2}$

→ Hence statement I is false and II is right

31. The transfer function of a phase lead compensator is given by $D(s) = \frac{3\left(s+\frac{1}{aT}\right)}{\left(s+\frac{1}{T}\right)}$.

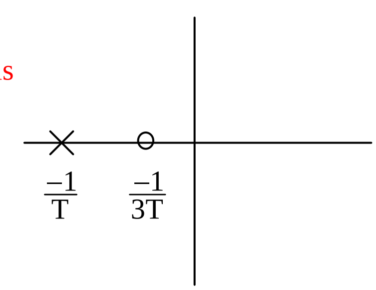
The frequency (in rad/sec), at which $\angle D(j\omega)$ is maximum, is

(A) $\sqrt{3T^2}$ (B) $\sqrt{\frac{3}{T^2}}$ (C) $\sqrt{3T}$ (D) $\sqrt{\frac{1}{3T^2}}$

Key: (D)

Sol: → It is given that transfer function of a lead compensator is

$$D(s) = 3\left[\frac{s+\frac{1}{3T}}{s+\frac{1}{T}}\right]$$



→ The frequency at which phase is maximum is given by geometric mean of pole, zero location,

$$\omega_m = \sqrt{\left(\frac{1}{3T}\right)\left(\frac{1}{T}\right)} = \sqrt{\frac{1}{3T^2}}$$

32. The voltage across and the current through a load are expressed as follows

$$v(t) = -170\sin\left(377t - \frac{\pi}{6}\right)V$$

$$i(t) = 8\cos\left(377t + \frac{\pi}{6}\right)A$$

The average power in watts (round off to one decimal place) consumed by the load is ______.

Key: (588.89)

Sol: It is given that

$$\rightarrow v(t) = -170\sin\left(377t - \frac{\pi}{6}\right)$$
$$= -170\sin(377t - 30°)$$
$$= 170\cos(377t + 60°)$$

$$\rightarrow i(t) = 8\cos\left(377t + \frac{\pi}{6}\right) = 8\cos(377t + 30°)$$

To calculate the phase difference between v(t) and i(t) both of them should in either +ve cos or +ve sin.

$$\rightarrow P_{avg} = |V_{rms}|\,|I_{rms}|\cos(\theta_v - \theta_I)$$
$$= \left(\frac{170}{\sqrt{2}}\right)\left(\frac{8}{\sqrt{2}}\right)\cos(60° - 30°)$$
$$= \frac{170 \times 8}{2}\cos(30°) = 680\cos 30° = 588.89 \text{ watts}$$

33. A DC-DC buck converter operates in continuous conduction mode. It has 48 V input voltage, and it feed a resistive load of 24 Ω. The switching frequency of the converter is 250 Hz. If switch-on duration is 1 ms, the load power is

(A) 12W (B) 6W (C) 48W (D) 24W

Key: (B or D) (IITM has given two answers)
NOTE: Perfect answer is only 24W.

Sol: Given

Resistive load $= 24\Omega$

$V_{in} = 48V, f_s = 250Hz, \ T_{ON} = 1ms.$

$T = \frac{1}{250} = 4ms.$

$D = \frac{T_{ON}}{T} = \frac{1}{4} = 0.25.$

As load is resistive and they did not mention current is ripple free. Hence the load power can be writer as.

$$P_{out}(load) = \frac{\left(V_{0(rms)}\right)^2}{R} = \frac{\left(\sqrt{0.25} \times 48\right)^2}{24}$$

$$= \frac{(0.5 \times 48)^2}{24} = \frac{24 \times 24}{24} = 24W$$

34. A single-phase fully-controlled thyristor converter is used to obtain an average voltage of 180V with 10 A constant current to feed a DC load. It is fed form single-phase AC supply of 230V, 50 Hz. Neglect the source impedance. The power factor (round off to two decimal places) of AC mains is _____.

Key: (0.78)

Sol: $V_{0(dC)} = 180V \ I_{dc} = 10A$

$V_s = 230V, 50\ Hz$

For, single- phase fully controlled thyristor converter if load is constant, then

$$V_{dc} = \frac{2V_m}{\pi}\cos\alpha = 180V$$

$$\cos\alpha = \frac{180 \times \pi}{2 \times 230\sqrt{2}} = 0.8692$$

$$IPF = \frac{2\sqrt{2}}{\pi}\cos\alpha = \frac{2\sqrt{2}}{\pi} \times 0.8692 = 0.782.$$

35. The closed loop line integral $\oint_{|z|=5} \frac{z^3 + z^2 + 8}{z + 2} dz$

evaluated counter-clockwise, is

(A) $+4j\pi$ (B) $-4j\pi$ (C) $+8j\pi$ (D) $-8j\pi$

Key: (C)

Sol: Let $F(z)=\dfrac{z^3+z^2+8}{z+2}$

$\therefore$ Singular point of $F(z)$ is $z=-2$; which lies inside $C:|Z|=5$.

Using Cauchy's integral formula, we have

$$\oint_C F(z)\,dz=\oint_C \frac{z^3+z^2+8}{z+2}dz=\oint_C \frac{z^3+z^2+8}{z-(-2)}dz$$

$$=2\pi j\left[z^3+z^2+8\right]_{z=-2}\left[\because \text{by cauchy's formula; } \oint_C \frac{f(z)}{z-z_0}dz=2\pi j f(z_0)\right]$$

$$\Rightarrow \oint_C \frac{z^3+z^2+8}{z+2}dz=2\pi j(-8+4+8)=8\pi j$$

36. A fully-controlled three-phase bridge converter is working from a 415V, 50 Hz, AC supply, It is supplying constant current of 100 A at 400 V to a DC load. Assume large inductive smoothing and neglect overlap. The rms value of the AC line current in amperes (round off tow two decimal places) is _____.

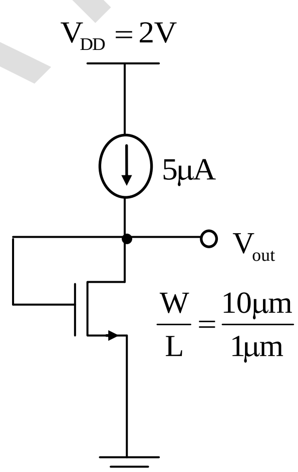
Key: (81.64)

Sol: The rms value of the AC line (for three phase bridge converter)

$$=\sqrt{2/3}\,I_{de}=\sqrt{2/3}\times 100=81.64\,A$$

37. The enhancement type MOSFET in the circuit below operates according to the square law.

$\mu_n C_{ox}=100\,\mu A/V^2$, the threshold voltage (V_T) is 500 mV. Ignore channel length modulation. The output voltage V_{out} is



(A) 2V (B) 100 mV (C) 500 mV (D) 600 mV

Key: (D)

Sol: Given $\mu_n cox = 100\mu A/V^2,\ V_t = 500mv, \lambda = 0$

The MOSFET is following square law, Hence it is operating in the saturation region

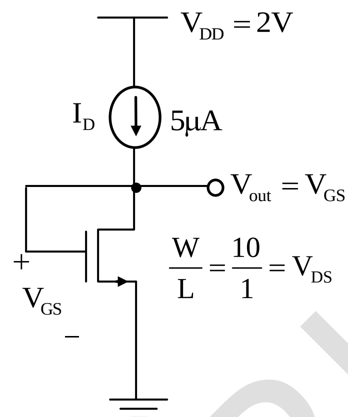
$$I_D = \frac{1}{2}\mu_n cox\left(\frac{W}{L}\right)(V_{GS} - V_t)^2$$

$$5\times10^{-6} = \frac{1}{2}\times100\times10^{-6}\left(\frac{10}{1}\right)(V_{GS} - 0.5)^2$$

$$(V_{GS} - 0.5)^2 = \frac{2\times5\times10^{-6}}{1000\times10^{-6}}$$

$$V_{GS} = 0.6 \text{ volt}$$

$$V_{out} = 600 \text{ mV}$$



38. In a 132 kV system, the series inductance up to the point of circuit breaker location is 50 mH. The shunt capacitance at the circuit breaker terminal is 0.05 μF. The critical value of resistance in ohms required to be connected across the circuit breaker contacts which will give no transient oscillation is_______.

Key: (500)

Sol: The critical value of resistance required to be connected across the circuit breaker contacts which will give no transient oscillation

$$R_{cr} = \frac{1}{2}\sqrt{\frac{R}{C}} = \frac{1}{2}\sqrt{\frac{50\times10^{-3}}{0.05\times10^{-6}}} = 500\Omega$$

39. The probability of a resistor being defective is 0.02. There are 50 such resistors in a circuit.

The probability of two or more defective resistors in the circuit (round off to two decimal places) is ______.

Key: (0.26)

Sol: Given,

the probability of a resistor being defective , i.e. $P = 0.02.$

$\Rightarrow$ Number of resistors, $n = 50.$

$$\therefore \lambda = np = 50\times0.02 = 50\times\frac{2}{100} = 1.$$

Let 'x' denote the number of defective resistors.

Using Poisson distribution, we have

$$P[x\ge 2]=1-P[x<2]=1-[P(x=0)+P(x=1)]$$
$$=1-\left[\frac{e^{-\lambda}\lambda^0}{0!}+\frac{e^{-\lambda}\lambda}{1!}\right] \qquad \left[\because P(x)=\frac{e^{-\lambda}\lambda^x}{x!}\right]$$
$$=1-e^{-\lambda}[1+\lambda]$$
$$=1-e^{-1}[1+1]=1-2/e\approx 0.26$$

40. The output expression for the Karnaugh map shown below is

RS \ PQ	00	01	11	10
00	0	1	1	0
01	1	1	1	1
11	1	1	1	1
10	0	0	0	0

(A) $Q\bar{R}+\bar{S}$ (B) $QR+\bar{S}$ (C) $Q\bar{R}+S$ (D) $QR+S$

Key: (C)

Sol:

$Q\bar{R}$

RS \ PQ	00	01	11	10
00	0	1	1	0
01	1	1	1	1
11	1	1	1	1
10	0	0	0	0

S

Output expression $=Q\bar{R}+S$

41. A periodic function $f(t)$, with a period of 2π, is represented as its Fourier series,

If $f(t)=a_0\Sigma_{\pi=1}^{\infty}a_n\cos nt+\Sigma_{n=1}^{\infty}b_n\sin nt$. .

$$f(t)\begin{cases}A\sin t, & 0\le t\le \pi\\ 0, & \pi\le t<2\pi'\end{cases}$$

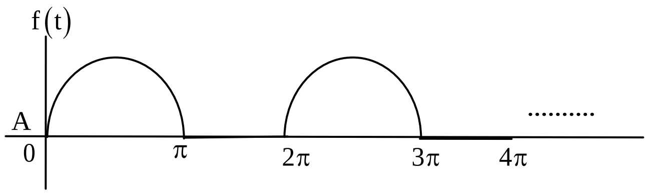
The Fourier series coefficients a_1 and b_1 of f (t) are

(A) $a_1=0; b_1=A/\pi$ (B) $a_1=\dfrac{A}{\pi}; b_1=0$

(C) $a_1 = \frac{A}{2}; b_1 = 0$ (D) $a_1 = 0; b_1 = \frac{A}{2}$

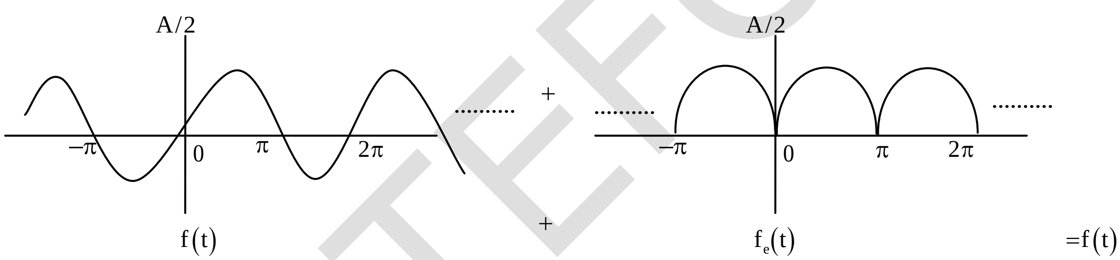
Key: (D)

Sol: As per the given description of f(t), if we draw its waveform, if looks like



→ One way to obtain its C.T.T.S is by obtaining its odd and even part and then by obtaining their individual C.T.F.S and finally we can add them to get complete C.T.F.S of f(t). However in this case we can pick the correct option by eliminating others.

$$\rightarrow \quad f(t) = f_o(t)1 + fe(t) = \left[\frac{f(t) - f(-t)}{2}\right] + \left[\frac{f(t) + f(-t)}{2}\right]$$



$$f_o(t) = \left[\frac{A}{2}\sin\omega_o t\right] + \sum_{n=1}^{N} a_n \cos\omega_o t$$

From this $b_1 = \frac{A}{2}$, So only option D satisfy this.

42. A 0.1 μF capacitor charged to 100 V is discharged through a 1 kΩ resistor. The time in ms (round off to two decimal places) required for the voltage across the capacitor to drop to 1V is________.

Key: (0.46)

Sol: It is given that

$\rightarrow$ since $V_c(\infty) = 0$ in this case

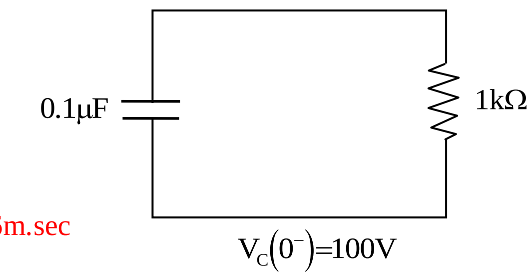
$V_C(t) = V_c(0^-)e^{-t/RC}$

$$V_C(t_x) = 100e^{-t_x/0.1\times1\times10^{-3}}$$

$$\Rightarrow 1 = 100e^{-10000t_x} \Rightarrow e^{-10000t_x} = \frac{1}{100}$$

$$\Rightarrow -10000t_x = \ell n(0.01)$$

$$\Rightarrow t_x = \frac{\ell n\,0.01}{-10{,}000} = 0.46\times10^{-3}\,\text{sec} = 0.46\text{m.sec}$$



43. A moving coil instrument having a resistance of 10Ω, gives a full-scale deflection when the current is 10 mA. What should be the value of the series resistance, so that it can be used as a voltmeter for measuring potential difference up to 100 V?

(A) 9990 Ω (B) 990 Ω (C) 99Ω (D) 9 Ω

Key: (A)

Sol: $V_m = 10\times10\times10^{-3} = 0.1V$

$$m = \frac{V}{V_m} = \frac{100}{0.1} = 1000$$

$$R_{Series} = (m-1)R_m = 999\times10 = 9990\,\Omega$$

44. A three-phase 50 Hz, 400 kV transmission line is 300 km long. The line inductance is 1 mH/km per phase, and the capacitance is 0.01 $\mu F/km$ per phase. The line is under open circuit condition at the receiving end and energized with 400 kV at the sending end, the receiving end line voltage in kV (round off to two decimal places) will be______.

Key: (418.59)

Sol: Given, line length = 300 km long (means long line).

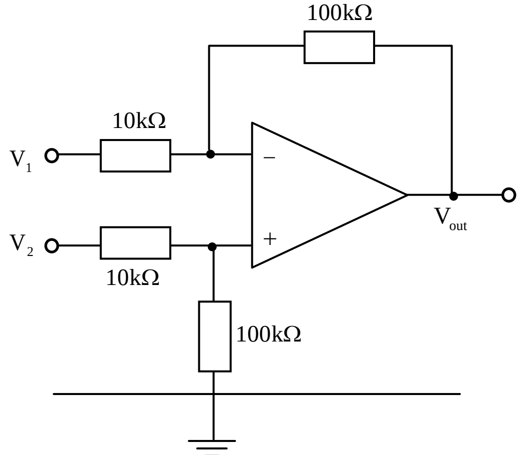
$V_S = 400\,kV(\text{line to line})$. $L = 1\,mH/km$ and $C = 0.01\mu F/km$.

$$V(\text{speed}) = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.01\times10^{-6}\times1\times10^{3}}} = 316227.766\,km/s.$$

$$A = 1-\frac{\beta^2}{2} = 1-\frac{\left(\frac{2\pi f\ell}{V}\right)^2}{2} = 1-\frac{\left(\frac{2\pi\times50\times300}{316227.766}\right)^2}{2} = 0.955.$$

$$V_{r(Noload)} = \frac{V_s}{A} = \frac{400}{0.955} = 418.59\,kV.$$

45. In the circuit below, the operational amplifier is ideal. If $V_1 = 10\,mV$ and $V_2 = 50\,mV$, the output voltage (V_{out}) is



(A) 100 mV (B) 600 mV (C) 400 mV (D) 500 mV

Key: (C)

Sol: $V_a = \frac{100}{100+10} \times 0.05$

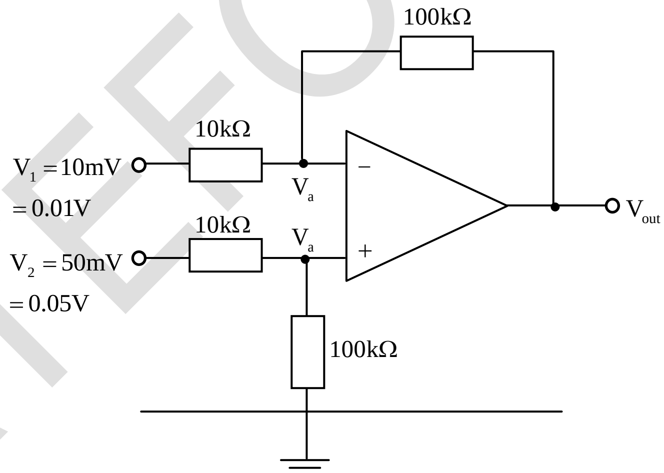
$V_a = \frac{1}{22}$ volt.

$\frac{V_a - V_{out}}{100} + \frac{V_a - 0.01}{10} = 0$

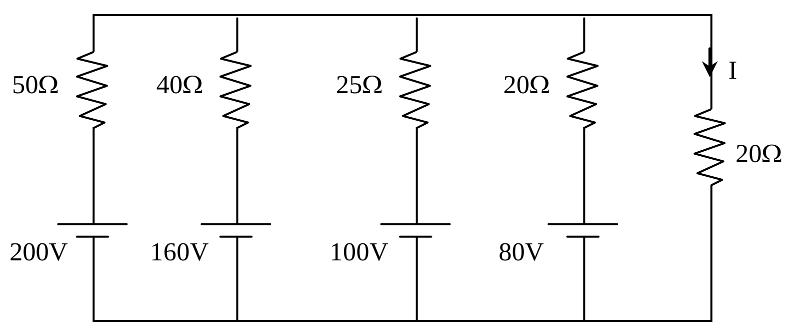
$V_a - V_{out} + 10V_a - 0.1 = 0$

$V_{out} = 11V_a - 0.1$

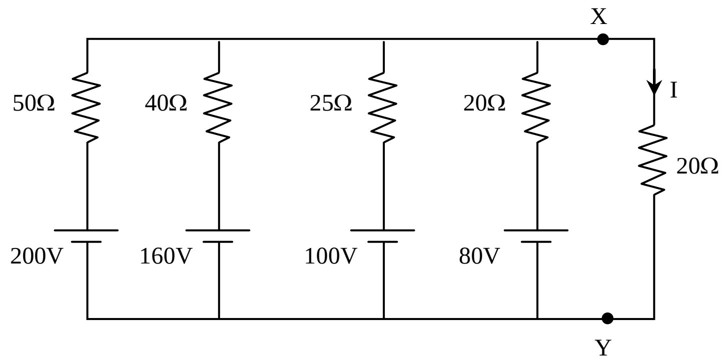
$= 11\left(\frac{1}{22}\right) - 0.1 = 0.4\text{ volt} = 400\text{ mV}$



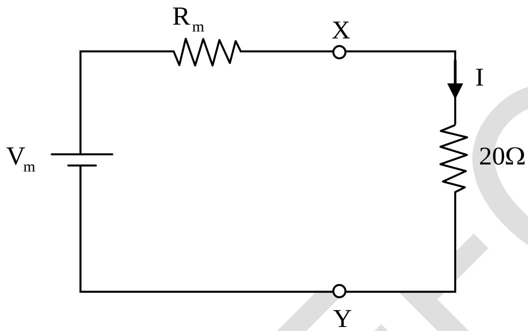
46. The current *I* flowing in the circuit shown below in amperes is ___________



Key: (0)



→ By Miliman's theorem, the network to the left of XY can be replaced by



$$I = \frac{V_m}{R_m + 20} \qquad \ldots(1)$$

where $$V_m = \frac{\left(200 \times \frac{1}{50}\right) + \left(160 \times \frac{1}{40}\right) - \left(100 \times \frac{1}{25}\right) - \left(80 \times \frac{1}{20}\right)}{\frac{1}{50} + \frac{1}{40} + \frac{1}{25} + \frac{1}{20}}$$

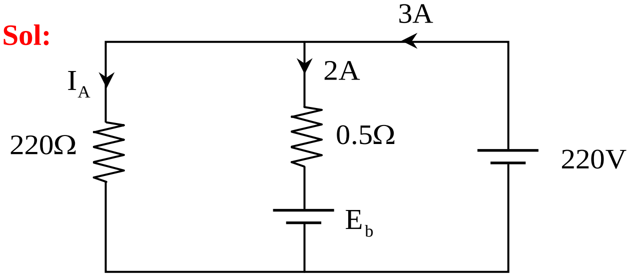
$$= \frac{4 + 4 - 4 - 4}{\frac{1}{50} + \frac{1}{40} + \frac{1}{25} + \frac{1}{20}} = 0$$

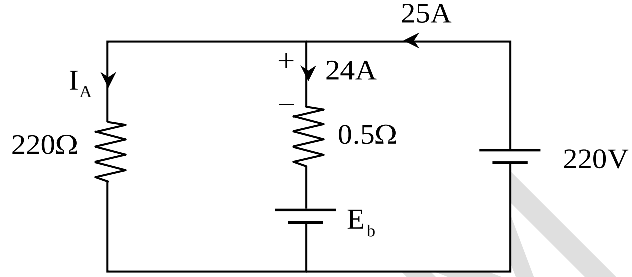
→ Putting V_m in equation (1) becomes $I = \frac{0}{R_m + 20} = 0A$

47. A 220V DC shunt motor takes 3A at no-load. It draws 25A when running at full-load at 1500 rpm. The armature and shunt resistances are 0.5Ω and 220 Ω, respectively. The no-load speed in rpm (round off to two decimal places) is _______ .

Key: (1579.32)

Sol:





No − load speed N_1

$$E_{b_1} = V - I_a R_a = 220 - 2 \times 0.5 = 219\text{Volt}$$

$N_2 = 1500$ r.p.m Full load

$$E_{b_2} = 220 - 24 \times 0.5 = 208\text{Volt}$$

$$\frac{N_2}{N_1} = \frac{E_{b_2}}{E_{b_1}} \times \frac{\phi_1}{\phi_2} \Rightarrow N_1 = \frac{N_2.E_{b1}}{E_{b2}} = \frac{1500 \times 219}{208} = 1579.32 \text{ rpm}$$

$\phi_1 = \phi_2$

48. In a DC-DC boost converter, the duty ratio is controlled to regulate the output voltage at 48V. The input DC voltage is 24V. The output power is 120W. The switching frequency is 50kHz. Assume ideal components and a very large output filter capacitor. The converter operates at the boundary between continuous and discontinuous conduction modes. The value of the boost inductor (in μH) is _________.

Key: (24)

Sol: Given, $V_o = 48V,\ V_{in} = 24V,\ P_{out} = 12000$

$f_{switching} = 50kHz,$

$$48 = \frac{24}{1-D}\left(\because V_{out} = \frac{V_{in}}{1-D}\right)$$

$$2 - 2D = 1,\ 2D = 1 \Rightarrow D = 0.5$$

$$P_o = V_o.I_o \Rightarrow I_o = \frac{120}{48}$$

$$R_{Load} = \frac{120}{48} = \frac{48 \times 48}{120} = 19.2\Omega$$

$$L_C = \frac{D(1-D)^2 R}{2f} = \frac{0.5 \times 0.5^2 \times 19.2}{2 \times 50 \times 10^3} = 24\mu H$$

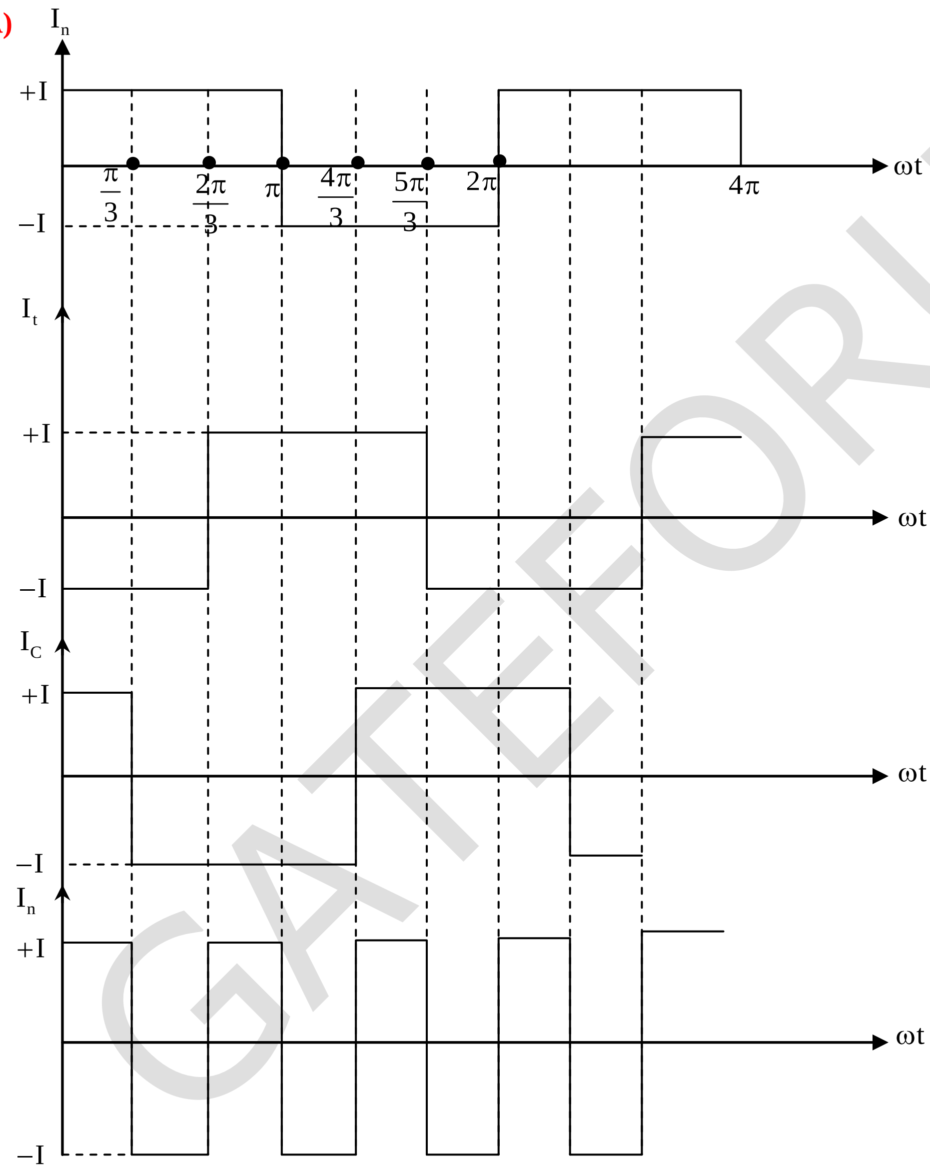
Hence, the converter operates at the boundary between continuous and discontinuous conduction modes, if the value of the boost inductor is 24μH.

49. The line currents of a three-phase four wire system are square waves with amplitude of 100A. These three currents are phase shifted by 120° with respect to each other. The rms value of neutral current is

(A) 100A (B) 0A (C) 300A (D) $\frac{100}{\sqrt{3}}$ A

Key: (A)

Sol:



Given 3-Ø, 4 wire system with line currents

I = 100 A (square wave)

$I_n = I_a + I_b + I_c$

From the I_n (Neutral current waveform)

$$I_{nrms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} I_n^2 d\omega t} = \sqrt{\frac{1}{2\pi}\left[2I^2\pi\right]}$$

$$I_{nrms} = I$$

Hence, $I_n = I = 100A$

50. A single-phase transformer of rating 25kVA, supplies a 12kW load at power factor of 0.6 lagging. The additional load at unity power factor in kW (round off to two decimal places) that may be added before this transformer exceeds its rated kVA is __________.

Key: (7.2)

Sol: Given, rating of transformer = 25kVA,

Existing load, $S = 12 + j16$

Let *P* is extra load with exceeding rated kVA.

$$(P+12)^2 + (16)^2 = 25^2 \Rightarrow P = 7.209kW$$

51. Consider a state-variable model of a system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\alpha & -2\beta \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \alpha \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Where *y* is the output, and *r* is the input. The damping ratio ξ and the undamped natural frequency ω_n (rad/sec) of the system are given by

(A) $\xi = \sqrt{\alpha};\ \omega_n = \frac{\beta}{\sqrt{\alpha}}$

(B) $\xi = \frac{\sqrt{\alpha}}{\beta};\ \omega_n = \sqrt{\beta}$

(C) $\xi = \sqrt{\beta};\ \omega_n = \sqrt{\alpha}$

(D) $\xi = \frac{\beta}{\sqrt{\alpha}};\ \omega_n = \sqrt{\alpha}$

Key: (D)

Sol: From the given state space model, we can say that

$$A = \begin{bmatrix} 0 & 1 \\ -\alpha & -2\beta \end{bmatrix},\ B = \begin{bmatrix} 0 \\ \alpha \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 \end{bmatrix},\ D = ??$$

In order to calculate ξ, ω_n. we need the transfer function of the system, which is given by

$$T(s) = C(sI - A)^{-1} B$$

$$= \begin{bmatrix}1 & 0\end{bmatrix}\left[\begin{pmatrix} s & 0 \\ 0 & s\end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -\alpha & -2\beta\end{pmatrix}\right]^{-1}\begin{bmatrix}0 \\ \alpha\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix} s & -1 \\ \alpha & s+2\beta\end{bmatrix}^{-1}\begin{bmatrix}0 \\ \alpha\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0\end{bmatrix}\left[\frac{1}{s(s+2\beta)+\alpha}\begin{bmatrix} s+2\beta & 1 \\ -\alpha & s\end{bmatrix}\right]\begin{bmatrix}0 \\ \alpha\end{bmatrix}$$

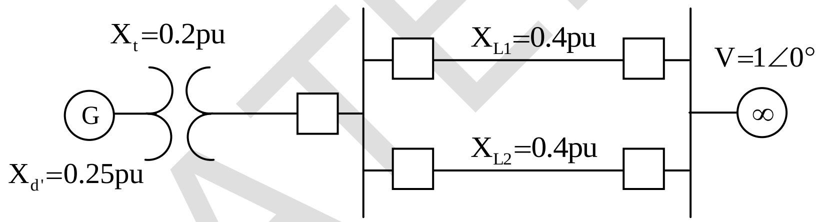
$$= \begin{bmatrix}1 & 0\end{bmatrix}\left[\frac{1}{s^2+2\beta s+\alpha}\right]\begin{bmatrix}\alpha \\ \alpha s\end{bmatrix} = \frac{\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\alpha \\ \alpha s\end{bmatrix}}{s^2+2\beta s+\alpha}$$

$= \dfrac{\alpha}{s^2+2\beta s+\alpha}$, by comparing with standard

$= \dfrac{\omega_n}{s^2+2\xi\omega_n s+\omega_n^2}$ We can say $\rightarrow$ $\omega_n = \sqrt{\alpha}$

$2\xi\sqrt{\alpha} = 2\beta$

52. In the single machine infinite bus system shown below, the generator is delivering the real power of 0.8 pu at 0.8 power factor lagging to the infinite bus. The power angle of the generator in degrees (round off to one decimal place) is _________.



Key: (20.5)

Sol: The generator is delivering the real power = 0.8pu at 0.8pf

$$I = \frac{0.8}{0.8} = 1\text{pu.} \quad X_{net} = 0.25+0.2+0.2 = 0.65\text{pu}$$

$$E_g = V + I_g Z = 1\angle 0 + 1\times 0.65\angle 90° - 36.86$$

$$E_g = 1\angle 0 + 0.65\angle 53.14 = 1 + (\cos 53.14 + j\sin 53.14)0.65$$

$$= 1 + 0.389 + j0.520 = 1.389 + j0.520$$

Hence load angle is 20.52°.

53. A 30kV, 50Hz, 50MVA generator has the positive, negative, and zero sequence reactances of 0.25pu, 0.15pu, and 0.05pu, respectively. The neutral of the generator is grounded with a reactance so that the fault current for a bolted LG fault and that of a bolted three-phase fault at the generator terminal are equal. The value of grounding reactance in ohms (round off to one decimal place) is _______.

Key: (1.8)

Sol: Given $X_1 = 0.25\text{pu},\ X_2 = 0.15\text{pu},\ X_0 = 0.05\text{pu}$

According to question, $LG = LLLG(\text{current wise}) \Rightarrow \dfrac{3}{0.25+0.15+0.05+3X_n} = \dfrac{1}{0.25}$

$$3X_n = 0.3 \Rightarrow X_n = 0.1\text{pu}$$

$$X_n = 0.1 \times \frac{30^2}{50} = 1.8\Omega$$

54. A 220V (line) three-phase,Y-connected, synchronous motor has a synchronous impedance of $(0.25 + j2.5)\Omega/\text{phase}$. The motor draws the rated current of 10A at 0.8 pf leading. The rms value of line-to line internal voltage in volts (round off to two decimal places)is ________.

Key: (245.35)

Sol: Given, V=220V, $V_{ph} = \dfrac{220}{\sqrt{3}} = 127.01\text{V}$

$$E = \sqrt{(V\cos\phi - I_a R_a)^2 + (V\sin\phi + I_a X_a)^2}$$

$$= \sqrt{(127.01\times 0.8 - 10\times 0.25)^2 + (127.01\times 0.6 + 10\times 2.5)^2} = 141.65\text{V}$$

$$E_{(\text{line})} = 141.65 \times \sqrt{3} = 245.34\text{V}$$

55. Consider a 2×2 matrix $M = [v_1 \ \ v_2]$, where v_1 and v_2 are the column vectors. Suppose $M^{-1} = \begin{bmatrix} u_1^T \\ u_2^T \end{bmatrix}$ where u_1^T and u_2^T are the row vectors. Consider the following statements:

Statement 1: $u_1^T v_1 = 1$ and $u_2^T v_2 = 1$

Statement 2: $u_1^T v_2 = 0$ and $u_2^T v_1 = 0$

Which of the following options is CORRECT ?

(A) Statement 2 is true and statement 1 is false

(B) Statement 1 is true and statement 2 is false

(C) Both the statements are false

(D) Both the statements are true

Key: (D)

Sol: Given

Let $M_{2\times 2} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, where $v_1 = \begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix}, v_2 = \begin{bmatrix} a_{12} \\ a_{22} \end{bmatrix}$

$$\Rightarrow M^{-1} = \frac{1}{a_{11}a_{22} - a_{21}a_{12}} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix},$$

$$\text{where } u_1^T = \frac{1}{a_{11}a_{22} - a_{21}a_{12}} \begin{bmatrix} a_{22} & -a_{12} \end{bmatrix},$$

$$u_2^T = \frac{1}{a_{11}a_{22} - a_{21}a_{12}} \begin{bmatrix} -a_{21} & a_{11} \end{bmatrix}$$

$$\therefore u_1^T v_1 = \begin{bmatrix} \frac{a_{22}}{a_{11}a_{22} - a_{21}a_{12}} & \frac{-a_{12}}{a_{11}a_{22} - a_{21}a_{12}} \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix} = 1$$

$$u_2^T v_2 = \begin{bmatrix} \frac{-a_{21}}{a_{11}a_{22} - a_{21}a_{12}} & \frac{a_{11}}{a_{11}a_{22} - a_{21}a_{12}} \end{bmatrix} \begin{bmatrix} a_{12} \\ a_{22} \end{bmatrix} = 1$$

$\therefore$ Statement-I is true.

$$u_1^T v_2 = \begin{bmatrix} \frac{a_{22}}{a_{11}a_{22} - a_{21}a_{12}} & \frac{-a_{12}}{a_{11}a_{22} - a_{21}a_{12}} \end{bmatrix} \begin{bmatrix} a_{12} \\ a_{22} \end{bmatrix} = 0$$

$$u_2^T v_1 = \begin{bmatrix} \frac{-a_{21}}{a_{11}a_{22} - a_{21}a_{12}} & \frac{a_{11}}{a_{11}a_{22} - a_{21}a_{12}} \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix} = 0$$

$\therefore$ Statement-II is true. So, both Statements are true.